TRIBUT
a Bicriterion Approach for Equilibrium Assignment

PTV AG
Stumpfstr. 1
D-76131 Karlsruhe
Tel.: +49-721-9651-0
Fax: +49-721-9651-299
Email: ptv@ptv.de

solutions for traffic.

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a Bicriterion Approach for Equilibrium Assignment

Authors:

François Barbier-Saint-Hilaire, INRETS
2, av. du Gén. Malleret-Joinville, F-94114 Arcueil, France
phone ++33/1/47407100, email bsh@inrets.fr

Markus Friedrich, PTV
Stumpfstr. 1, D-76131 Karlsruhe, Germany
phone ++49/721/9651/316, email markus.friedrich@ptv.de

Ingmar Hofsäß, PTV
Stumpfstr. 1, D-76131 Karlsruhe, Germany
phone ++49/721/9651/312, email ingmar.hofsaess@ptv.de

Wolfgang Scherr, PTV
Stumpfstr. 1, D-76131 Karlsruhe, Germany
phone ++49/721/9651/324, email wolfgang.scherr@ptv.de

Abstract:

TRIBUT is a bicriterion traffic assignment method which equally considers travel time and cost. The trip choice between different paths is modeled by defining the value of time as a random variable with a distribution of the log-normal type, thus considering that each trip has a specific willingness to pay toll for travel time reduction. Numerous applications of TRIBUT in Europe, Asia and North America have shown that this approach offers a significantly better price elasticity than monocriterion methods. The paper discusses the theoretical framework of TRIBUT in its most prominent features, i.e. randomly distributed values of time, the principles of path search and path choice. Furthermore it presents different aspects of the application in practice, in particular the definition of different demand classes, the modeling of linear or non-linear pricing schemes and the value of time estimation.

Keywords:

Bicriterion traffic assignment, toll modeling, value of time, equilibrium assignment

1 INTRODUCTION

1.1 General Overview

In the modeling of toll roads, two criteria have to be considered during assignment: path time and path cost, i.e. toll. The conventional approach of static toll assignment simplifies this bicriterion problem to a monocriterion problem by using a constant value of time for the path search, which basically converts time and cost into generalized impedance. Consequently there is no difference between toll and non-toll assignment, since both use the conventional monocriterion path search algorithms implemented in most existing transport modeling packages. In contrast to this conventional method, this paper will present the specific assignment procedure TRIBUT which is based on a bicriterion path search algorithm. Bicriterion assignment means, that the two criteria time and cost are evaluated and stored separately during path search and path choice. The individual choice between different paths is modeled using random distributed values of time, that can be defined specifically for different person groups.

1.2 History of TRIBUT and Related Work

TRIBUT has been developed by the French research institute INRETS and was implemented by François Barbier-Saint-Hilaire [1] within the French assignment program DAVIS. Since the early 1990's TRIBUT has been used by French transport engineers in numerous toll projects throughout the world (see LEURENT [2]). This French development initiative is not surprising, as France was the first European country to introduce a comprehensive toll scheme for its motorway network. In this context, a large number of toll motorway projects manifested the need for more sophisticated models evident and French research became leading in bicriterion assignment models as well as in the empirical research on user behavior concerning road pricing.

With the end of 1998, TRIBUT got a “new look” within the transportation model VISUM [3, 4] which is part of the commercial software package PTV-VISION. This is the result of a two year software development period performed by INRETS together with the German software company PTV. As a result TRIBUT is no longer based on the Frank/Wolfe global equilibrium approach but has inherited the o-d-based equilibrium from VISUM’s former monocriterion procedures.

In 1995 DIAL [5] published a fundamental paper on bicriterion assignment which is close to the TRIBUT approach except that he permits time *and* cost to be flow-dependent. We refer to this work as it was very helpful for us in finding the appropriate English words and definitions.

1.3 General Problem Formulation

For toll assignments, the criteria for choosing path p consist of time t_p and cost c_p. The objective function or the generalized path choice criterion $Crit_p$ can be formulated as follows:

$$Crit_p = t_p + c_p / VT = \sum_{L \in p} t_L + \left(\sum_{L \in p} c_L \right) / VT$$

with:

t_L	travel time on a network object L as a function of traffic volume $t_L = t(vol_L)$, L may represent a link, a node or a turning movement,
vol_L	volume of link L,
c_L	toll value for using link L, assumed to be invariant of link volume,
VT	value of time in [e.g. $/h] .

It is assumed, that each individual trip tries to minimize this criterion $Crit_p$ in its path choice within a road network. The way $Crit_p$ is formulated above, it represents a generalized time. $Crit_p$ may as well be defined as generalised cost by using the value of time VT for converting time into cost, which will lead to identical deductions.

So far the conventional monocriterion toll assignment and TRIBUT can be described by the same objective function. However, they differ in modeling the value of time VT:

- In the *monocriterion* approach the value of time VT is assumed to be constant for all trips, or at least for all trips within one trip class, i.e. o-d matrix. Therefore the expression c_p/VT in the objective function $Crit_p$ represents a constant supplement to time t_p for each path p. As a consequence the value of $Crit_p$ is identical for all trips.
- In the *bicriterion* approach of TRIBUT the value of time VT is randomly distributed. That way each trip within a matrix can apply a specific value of time and it is taken into account that each trip has its own perception about spending time and money for travel.

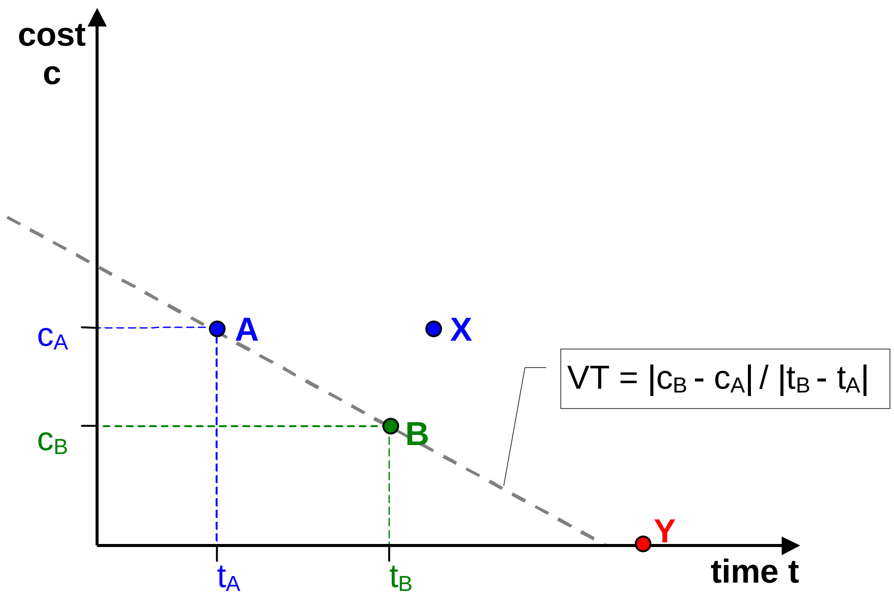
This assumption of individual values of time has various consequences on the model structure:

- During assignment both criteria t_R and c_R need to be accessible for each path at all times, so the values for time and cost must be stored in the data structures.
- A unique "best" path does not exist, as each trip has its specific "best" path. This has two effects: (1) the assignment will produce more alternative paths, (2) a multi-path-search algorithm is required.

1.4 The Time-Cost Diagram

The time-cost diagram displayed in Figure 1 may illustrate some aspects of the bicriterion path choice. In this diagram alternative paths for one o-d pair are represented by a set of points, each characterized by time and cost, e.g. path A = (t_A,c_A). A specific value of time VT can be represented by any straight line with the slope -VT. If the same straight VT line encounters two paths, they are considered indifferent, i.e. ”equally good” for a user who has this particular VT. This VT, which is determined by two alternative points, is also described as “critical value of time” for the two alternatives in question. In Figure 1 the straight line representing the critical VT for A and B is drawn as a dotted line.

Figure 1: Time-cost diagram with alternative paths and critical value of time



The point where the VT-line fits the cost-axis represents the total cost equivalent of both paths A and B, for the specific value of time, i.e. the critical value of time. The corresponding point on the time-axis represents the general criterion (i.e. the time equivalent). It is obvious that trips will prefer A to X for any value of time, i.e. A dominates X. More generally for any given VT, each path located to the right side of the VT-line is dominated by A and B (so is Y), because it is assumed that all trips prefer paths that minimize the general criterion as well as the cost equivalent.

2 THE TRIBUT MODEL AND ALGORITHMS

2.1 Value of Time as a logN-distributed Random Variable

As mentioned before, it is assumed that each vehicle applies its individual value of time VT. This assumption is reflected in the model by defining the value of time VT as a random variable with a distribution of the log-normal type:

$$VT = \log N\left(\overline{vt}, \sigma\right)$$

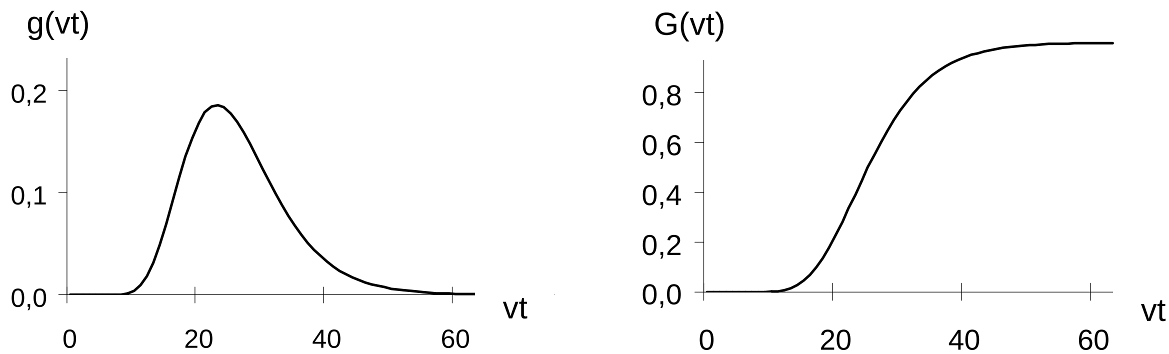
with the following distribution parameters:

$\overline{vt}$ — the median of $VT = \log_N\left(\overline{vt}, \sigma\right)$

σ — the standard deviation of the associated random variable $Y=\log_e(VT)$, where Y is normally distributed.

The logN-distribution is widely used in income statistics. One important property of the logN-distribution for income or toll modeling is, that the probability equals zero for negative values, which is a trivial assumption for values of time. The use of the median $\overline{vt}$ as positioning parameter may appear strange, but this corresponds to a convention in income statistics to publish quantils rather that mean values. Nevertheless the logN-distribution could as well be defined by the mean $\mu=\log_e(\overline{vt})$ and standard deviation σ both of the associated normal distribution $Y=\log_e(VT)$.

Figure 2: Density $g(vt)$ **and distribution function** $G(vt) = \int_{-\infty}^{vt} g(vt)$ **for** $VT = LogN(25;0.3)$



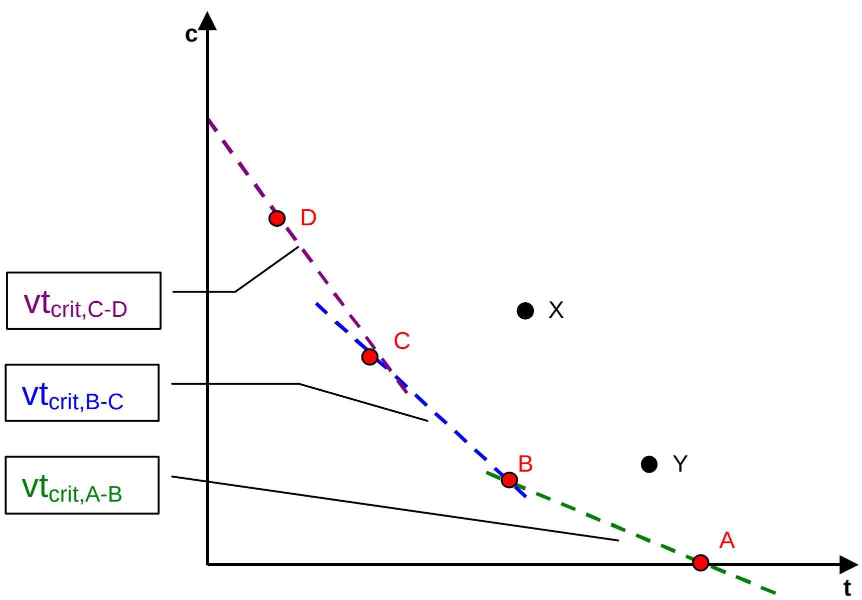
Using TRIBUT in a planning project, the definition of the VT distribution, i.e. of the two parameters, is one of the most crucial steps. Chapter 3.3 will therefore discuss empirical methods to determine the distribution parameters and present latest research results carried out in Europe.

2.2 Path Search: Efficient Frontier as Exclusive Criterion

In monocriterion methods one unique best path (shortest path) can be determined for each o-d pair. In the case of TRIBUT, however, due to the random distribution of VT, numerous "best paths" must be determined and stored in memory not only during path search but during the entire assignment procedure. That is the reason for the high complexity of multicriterion methods. Nevertheless, it is possible to reduce this complexity by identifying the *efficient* paths for each o-d pair.

Figure 3 shows a path search with six paths, where paths A, B, C and D represent the set of efficient paths. It can easily be demonstrated either graphically or analytically that there is no VT for which paths X or Y would be preferred over A, B, C or D. Generally speaking, the convex curve formed by the three critical VT-straight-lines A-B, B-C and C-D limits the range of relevant cost-time combinations to the right side. This convex curve is therefore called efficient frontier. Therefore X and Y can be deleted in the example.

Figure 3: Critical values of time building up the efficient frontier



The introduction of the efficient frontier has important consequences:

- Only the efficient paths need to be stored during path search for the subsequent path choice. As a consequence the majority of the various possible paths for one o-d pair can be discarded thus limiting computing time and memory.
- Nevertheless TRIBUT needs to perform a simultaneous multi-path search that is more complex than the best-path-search procedures in the monocriterion case.
- The set of efficient paths of any o-d pair is unique for one state of the network, meaning that it does not depend on the kind of VT-distribution which has been defined. Hence in the case of a multi-class assignment, only one search step has to be performed in each equilibrium iteration, although the subsequent demand allocation will be determined specifically for each demand class.

The TRIBUT multi-path-search algorithm consists of two steps:

1. First, it uses a classic shortest path search to determine the paths with minimum travel time from one origin o to *all network elements*. Note, that a classic shortest path search would only build up paths from one origin to *all destinations*. This first step is not sufficient to determine the entire efficient frontier, but it will find the fastest paths, which will definitely be the most expensive paths on the efficient frontier.
2. Next, the procedure extends the path-tree with less expensive paths. This expansion is achieved not by considering the travel time, but by considering the critical value of time between the last inserted path to a network object and the other candidates.

2.3 Path Choice for a Given Set of Alternatives

Path choice aims to distribute the travel demand of an o-d pair onto the set of available paths. In the bicriterion case, the demand is allocated to the efficient paths. The way the total o-d demand is distributed onto the different paths, depends on the critical values of time between two adjacent paths along the efficient frontier. In our example, there are three critical VTs defined by the path pairs A-B, B-C and C-D. The demand share of each efficient path is deduced from the given probability distribution function.

In Figure 4 the distribution function is evaluated for the three values $VT=vt_{crit,A-B}$, $VT=vt_{crit,B-C}$ and $VT=vt_{crit,C-D}$. The share P(A) of alternative A on the lowest cost level is:

$$P(A) = G\ (VT = vt_{crit,A-B})$$

The shares of B, C and D are:

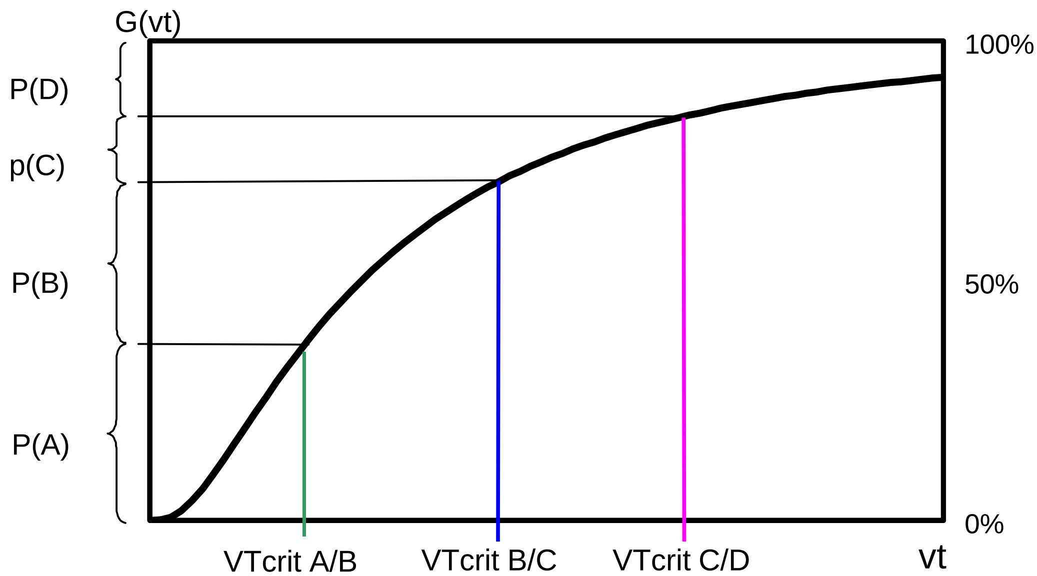
$$P(B) = G\ (VT = vt_{crit,B-C}) - P(A)$$

$$P(C) = G\ (VT = vt_{crit,C-D}) - G(VT=vt_{crit,B-C})$$

$P(D) = 1 - G\ (VT = vt_{crit,C\text{-}D})$

This allocation states, that trips, performed by travelers who are not willing to spend money on travel (VT≈0), up to trips with a VT <= $vt_{crit,A\text{-}B}$ will definitely choose the cheapest path A. Traveler on the more "wealthy" side of the trip demand with VT >= $vt_{crit,C\text{-}D}$ will choose the fastest and most expensive path D.

Figure 4: Path choice for initial loading



Within the TRIBUT assignment this kind of path choice is applied only once to provide an initial loading. This step which considers each o-d pair independently, is similar to an "all or nothing" assignment and provides an initial solution. But as travel time on network objects, i.e. links and nodes, is capacity restraint-dependent, the path choice for one specific o-d pair depends on the path choice of all other o-d pairs. This leads to an iterative procedure attempting to find a solution, where all o-d pairs are in equilibrium state.

2.4 Adjustment of Path Choice During the Equilibrium Iteration

An o-d pair is in an equilibrium state, if the following conditions are accomplished:

- Path search does not find another efficient path for the o-d pair.
- The flow-dependent travel time is identical for all efficient paths on the same cost level.
- The shares of demand on the different cost levels correspond to the VT-distribution.

TRIBUT performs a path search at the beginning of each new assignment iteration. If new paths are found which are located on the efficient frontier or to the left of it, they are added to the set of existing efficient paths. VISUM stores all path-information (itinerary, used network objects, allocated demand), so that complete path information is available during the whole assignment process as well as after assignment for post-assignment analysis.

If new paths are found and stored for a specific o-d pair, the o-d demand must be reallocated within the resulting new efficient frontier to obtain a new equilibrium state. This is achieved in two steps:

- INTRA-Level-Balancing: reallocation of demand among the paths of the same cost level of the o-d pair and recalculation of flow-dependent time on the associated paths and their links,
- INTER-Level-Balancing: reallocation of demand between the paths of two adjacent cost levels and recalculation of flow-dependent time.

It is important to note, that during the balancing process of shifting demand from one efficient path to another, path travel time changes, as it depends on the traffic volumes. So during balancing, the shape of the efficient frontier is modified and the critical values of time change.

Figure 5 illustrates an efficient frontier with three paths A, B, C. If a new path N is found which shows the same cost level as the efficient path B, demand is shifted from B to N. As a consequence travel time t_B and t_N change. That way both paths will move to a point in the middle of their initial positions. As a result the critical values of time for the adjacent cost levels will change. So the demand must be reallocated among all efficient paths and between the different cost levels. Note, that this simple case of adjustment only occurs if the paths A and C do not share links with the B or N, so that t_A and t_C are not affected.

Figure 5: Adjustment of the efficient front during INTRA-Level-Balancing

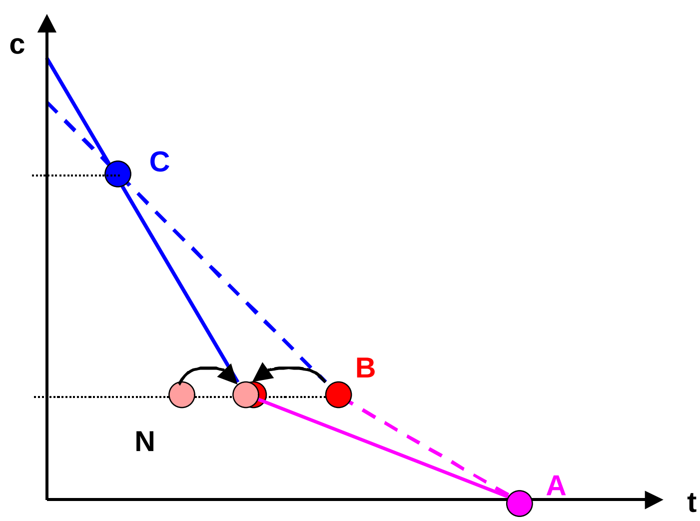


Figure 6 presents a simplified flow chart of the TRIBUT assignment

Figure 6: Flow chart of the equilibrium iteration

```
** INITIAL LOADING: **
FOR origins o := 1 ... N DO
    Perform MULTI-PATH-SEARCH to all destinations d := 1 ... N
    FOR all destinations d := 1 ... N DO
        Allocate o-d-trip demand on the efficient paths of o-d
        Store the efficient paths of o-d
    Compute flow dependent link times for all network objects of the efficient path set

** EQUILIBRIUM ITERATION: **
WHILE NOT all-od-in-equilibrium DO
    all-od-in-equilibrium := TRUE
    FOR origins o := 1 ... N DO
        Perform MULTI-PATH-SEARCH to all destinations d := 1 ... N
        IF new efficient path are found THEN o-in-equilibrium := FALSE
        Store paths
        WHILE NOT o-in-equilibrium DO
            o-in-equilibrium := TRUE
            FOR all destinations d := 1 ... N DO
                WHILE NOT od-in-equilibrium DO
                    od-in-equilibrium := TRUE
                    FOR all cost-levels of o-d DO
                        INTRA-Level-Balance
                        Compute link times for all network objects
                    FOR all neighbor-cost-levels of o-d DO
                        INTER-Level-Balance
                        Compute link times for all network objects
                    IF flow has been changed on any network object THEN
                        od-in-equilibrium := FALSE
                        o-in-equilibrium := FALSE
                        all-od-in-equilibrium := FALSE
```

The flow-dependent travel time on a link is calculated at three points in this procedure:

- In the initial loading, after each origin computation.
- In the INTRA-Level Balance, when vehicles are shifted from one path to another within the same cost level. For each modified link volume, the actual link time has to be computed again.
- In the INTER-Level Balance, when vehicles are pushed from one cost level to another and shifted between the paths of those levels.

3 THE APPLICATION OF TRIBUT IN PRACTICE

3.1 Toll Schemes

VISUM allows to define linear and non-linear road pricing schemes. Linear toll is modeled as a toll value per road segment (link). To model classical toll schemes, where toll is typically linear to trip length and where toll is paid for the next road section at stations which are located at entry or exit points, a linear toll model is sufficient. In modern toll projects, more sophisticated systems of pricing are applied. Especially telematic techniques allow to design price schemes, where the price to drive from A to C via B may not be equal to the sum of A-B and B-C. These non-linear toll schemes can be modeled within VISUM as a price matrix between motorway entries and exits.

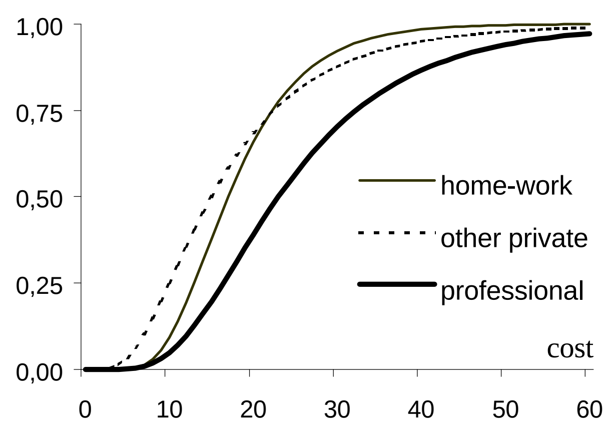
3.2 Simultaneous Multi-Class Assignment

VISUM provides multi-class assignment, where several demand classes with specific o-d matrices can be assigned simultaneously.

The introduction of demand classes to the model is helpful in the case of group specific pricing for different vehicle types or travelers with different types of tickets (single riders, commuter pass holders).

Additionally specific VT-distributions can be defined for each demand class thus leading to a more realistic composition of the entire demand and of its elasticity to pricing policies. An example of user classes for home-work-commuters, other private demand and professional demand is shown in Figure 7.

Figure 7: Multi-class VT-distributions



3.3 Finding the parameters $\overline{vt}$ and σ

Applying TRIBUT in a planning project, the definition of the VT-distribution, i.e. of the two distribution parameters $\overline{vt}$ and σ of the value of time distribution for different trip classes – or unique ones for a global travel demand – is one of the most crucial steps. In practice there are three ways to find the parameters: revealed preference surveys, stated preference surveys or macroeconomic calculus. The weakest but cheapest method is the macroeconomic calculus which for instance divides total work income of a society by the total work time in order to obtain a mean value of time for trips to work. Such an approach does not reflect real behaviour, e.g. the significant differences in the behavior of commuters in urban areas compared to long distance travelers. Secondly, this approach determines a *mean* value of time, which is less appropriate than quantiles (medians).

When toll is introduced into a society without revealed toll experience, usually stated preference methods are applied. In the stated preference interview different hypothetical situations with variation of time and cost are simulated to find the critical cost-time combination ("transfer price"), where the traveler changes his behaviour. Maximum likelihood estimation will determine the two logN-parameters from a stated or revealed preference sample. It has been found that persons can hardly imagine a toll situation as it is proposed in a stated preference interview if they are not used to toll in real life. Consequently, the standard deviation σ is mostly overestimated by the stated preference method.

The best method is to estimate the value of time on the basis of revealed preferences. This requires a toll road and pricing system, which operates at least three or four years prior to the survey, to ensure that the system has come to an equilibrium. Recently a revealed preference study has been carried out in the area of Marseille/France [6]. The method consisted of an o-d survey, measuring the shares of cars using the toll road or the alternative non-priced infrastructure and real travel time for each o-d at different day periods. The most important results are:

- Time saving was found to be the most explicative variable of the use of toll roads, no statistical effect was found for travel time deviations or road comfort.
- It was found that the median of the value of time is a very significant and stable value, even if different types of distributions are estimated.
- The logN parameters for the global demand were estimated by $\overline{vt}$ = 57 FF/H ≈ 10 Euro/h and σ = 0.66.
- The different trip purposes show a less significant influence on the value of time parameters than the fact whether a traveler must pay the toll "out of his own pocket" or whether he gets the toll refunded for instance by his company.

3.4 Some Aspects of Travel Time Modeling

As cost is not flow-dependent, it is a given model input. On the other hand, time is modeled by the help of flow-dependent functions and therefore represent a less certain input of the assignment model. But the quality of the forecasted volumes in toll projects depends largely on path time and related cost. Therefore the modeling of the flow-dependent time on links and nodes requires more attention than in ordinary planning projects without toll. The modeler has to consider especially the following aspects:

- For trips which originate or terminate outside the scope of the model, only a part of their path is covered by the model. Therefore it is not possible to evaluate the total travel time and thus it is not recommended to apply the same value of time distribution as for internal trips.
- Link and node flows that exceed capacity should be avoided. In this case capacity restraint functions do not produce realistic travel times. Especially if peak hours are modeled, the capacities of highly charged links need to be defined very carefully.

4 CONCLUSION

TRIBUT has been used for several road toll studies especially within urban areas throughout Europe, America and Australia. With its implementation in VISUM, TRIBUT is available for practitioners on a multi-language platform. Based on the experience of successful studies, TRIBUT is widely accepted by road financing companies to establish realistic revenue forecasts and by planning authorities to estimate the impacts on the complementary network. The advantages of the TRIBUT method become obvious when different pricing policies need to be tested. The conventional monocriterion approach turned out to be not appropriate to model any realistic price elasticity of demand regarding price variations.

Recent empirical research in Europe confirms the TRIBUT approach concerning the distribution of the VT as log-normal [6]. To improve the exploitation of bicriterion network models, empirical research is needed in the following areas:

- Revealed preference surveys: to determine specific value of time distributions for different societies, different city areas and for different demand classes.
- Transferability of values of time: how can distributions which have been estimated for one city or state be applied to other study areas? How can differences in income and wealth be taken into account when parameters are geographically transferred?
- Usage of stated preference based parameters: Applying stated and revealed preferences for the same study area may help to identify systematical errors of the stated preference method and to establish formulas offering realistic value of time distributions for projects where only stated preference surveys could be carried out.

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