Static Assignment versus Dynamic Simulation

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solutions for traffic.

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1 INTRODUCTION

Mobility is a prerequisite of modern society and economy. 80% of the overall transport of people (measured in person kilometers) in Germany were made on roads. Well designed road networks are emminent for private car transport as well as public transport by buses. At a time of growing concern about the extra pollution due to road transport well designed road networks which minimize land use and still sustaining easy accessibility of all destinations are required. Mathemetical models are applied for the networks to locate bottlenecks and to find the most cost-effective alternatives. This paper will indicate some of the models applied by practioners and outline its mathematical backgrounds.

In the second chapter some basic ideas of equilibrium assignment will be explained. Chapter three will indicate the underlying theory of macroscopic traffic flow simulation and the fourth chapter outlines concepts of microscopic traffic flow models. Readers who wish to get a deeper insight are referred to the textbooks (Bell, 1997; Sheffi, 1985). Some of the content of this paper is taken from those books without citation at each occurrance.

2 ASSIGNMENT MODELS

2.1 Mathematical background

In transportation literature assignment is called the process to distribute a given amount of demanded trips on a network of links. In empty networks all travellers from one origin to one destination would use the shortest path if all would have the same knowledge about the length of each path. As congestion builds up on the shortest path, some travellers will divert to less occupied paths.

A road network is represented by a graph consisting of nodes and arcs (directed links). The travel demand starts at a centroid of a zone (origin) and ends at a centroid of another zone (destination). A centroid is connected via one or more nodes to the network. Nodes represent junctions. For modelling purposes junctions are sometimes decomposed in up to 4 nodes and 12 links for a four-leg junction to model each turning movement by a separate link.

It is assumed that link costs exist for each directed link and sometimes even junction costs to reflect delay of turning manouvers. Link cost might be a linear combination of travel time, comfort, toll fees and distance travelled but the first one usually being dominant. Link travel time depends on the link length l, the free flow speed v_0, the volume per unit time q and the capacity cap (maximum possible volume). Within road networks the link cost function is monotonically increasing as q is increasing. For computational reasons differentiable link cost function are favoured; the most prominent one being of the type

$$t_a(q) = \frac{l}{v_0} \cdot \left(1 + a \left(\frac{q}{g \cdot cap} \right)^b \right)$$

whereas t_a represents the time travelled on link a and a, b and g being parameters to define the steepness of the volume-delay function.

Link cost functions of public transport networks in contrast reflect the transit time table, which usually does not consider explicitly the current traffic volume. Therefore link cost for public transport is quite often a constant up to a certain maximum capacity.

A path is a sequence of directed links. The cost of a path is the sum of the costs of all links comprising the path. Assume that all travellers have full information about the cost of each path and they behave rational as all try to minimise their path cost. At the user equilibrium the cost of travelling between origin i and destination j will be the same on all used paths. None of the unused paths will cost less. This user equilibrium is referred to Wardrop´s 1st principle. The 2nd principle states a system equilibrium which mimises the overall cost of the paths of all origin-destination pairs. However since individuals rather behave according to the 1st principle it is applied most widely in transportation science.

In the case of two possible paths between i and j the user equilibrium can be solved graphically. In Figure 1 the path costs for route 1 and route 2 are shown. Assume that there is a demand of 1000 vehicles from i to j. Route 1 being much shorter but also having less capacity and a lower free flow speed. The travel time of route 2 is drawn as $t_a(1000-q)$ since the remaining of the 1000 travellers will take route

2. The curves intersect at q=450 trips on route 1 and 550 on route 2.

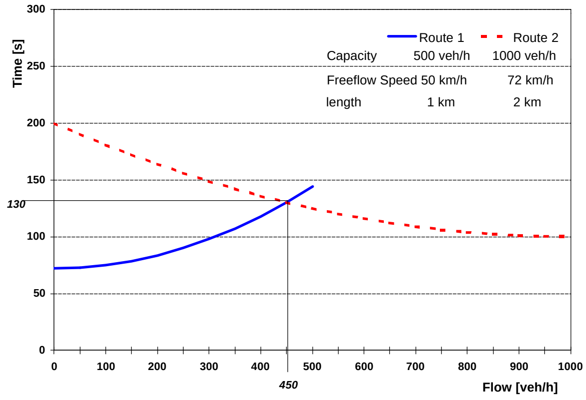

Fig. 1. Link cost functions of two routes both using α=γ=1 and β=2 and a demand of 1000 trips on both routes

In all practical applications a graphical solution is impossible but methods of mathematical programming are well suited. For reasons of simplicity the link cost function is assumed to be independent of other links, which not necessarily holds true in urban networks with semi-compatible turning movements. The link cost function is then called to be mathematically separable. Then the objective function of the user equilibrium can be written as

$$\min z(\mathbf{x}) = \sum_{a=1 \text{ to } N} \int_{x=0}^{x=q_a} t_a(x)\, dx$$

where N is the number of links and q_a represents the flow on link a. The equilibrium assignment problem is to find the set of link flows **x**, that satisfy the above minimization problem and the demand of trips of all origin-destinations is satisfied. Since the link cost function t_a is monotonically increasing, the Hessian matrix $\nabla^2 z(\mathbf{x})$ is positive definite and an efficient convex combination method from mathematical programming can be applied to solve the above minimization problem.

Refinements of the above well known objective function were presented by various authors to model reality more realistically. The assumption that all travellers have perfect information about link travel time at any level of congestion is relaxed in stochastic user equilibrium assignment methods. These models use concepts of utility maximization taking logit or probit functions to define the path choice. The solutions for the stochastic user equilibrium are not as straightforward as for the above deterministic user equilibrium since the objective function and the derivatives are formulated in terms of paths and not only links. Details and a comprehensive list of references is given in Bell (1997) and Sheffi (1985).

Recent advancements for real world problems deal with multimode assignment. Origin-destination matrices are forecasted for each mode of travel (car, public transport). After assigning the trips by mode on the network, impedances such as travel time and trip cost are computed. These impedance values are input values for a next iteration of the mode choice model. Logit functions are commonly applied to estimate the probability of each mode.

Depending on the trip purpose (trips to work, business related trips, leisure trips, shopping trips etc.) travellers might experience different link cost functions. Travel time is by far the most important criteria for business trips while the out-of-pocket trip related expenditure for leisure trips is important as well. Recent assignment models deal with multiple link cost functions (Leurent, 1996).

2.2 Applications of equilibrium assignment methods

Typical applications of equilibrium assignment deal with the topology of road and public transport networks.

- Schemes to close roads and traffic calming (deleting links respectively reducing free-flow speed and capacity of certain links)
- Construction of new roads and rail links such as new bridges, tunnels and bypasses (adding new links)
- Increase of the design speeds on roads and rail links by widening roads and new tracks (increasing capacity and free-flow speed of certain links)
- Reduction of travel times on certain links by longterm improvement of travel control at junctions with observed continous bottlenecks (reducing delay times at junctions for certain movements)
- Introduction of new public transport services to improve the accessibility of certain spatial areas (adding new transit lines on existing links or even adding new links)
- Improvement of public transport service by additional courses of existing transit lines (increase the service rate (eg. shorter headways) and passenger capacity of that line which will improve the attractiveness through reduced average waiting times)

The time horizon usually ranges from longterm (forecast period 5 til 25 years) to mediumterm (1 to 5 years). Transport planners usually set up various network topologies and compare the assignment results by impact analysis considering cost and benifits.

Various software products are applied by transport planners at consultancies, public agencies and municipalities to solve these tasks. The following network examples are all run using the software VISUM. Typical examples deal with networks of considerable size.

Application	zones	nodes	links
Road&transit network of Leipzig	507	2.580	6.786
Regional Traffic Master Plan Stuttgart	780	8.410	20.800
Road network of Cologne taken from GIS-data	815	15.098	39.924
Chicago Regional Plan	1790	20.405	39.320
Federal transport plan of Germany	402	36.642	79.800

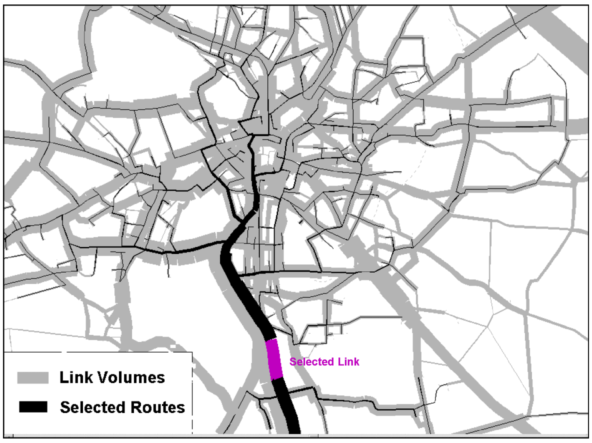

Fig. 2. VISUM plot depicting link volumes and traffic using the marked selected link superimposed

3 MACROSCOPIC TRAFFIC FLOW SIMULATION

3.1 Mathematical background

While assignment models distribute an origin-destination matrix for one period of time all at once, traffic flow simulation models move vehicles through a network. Macroscopic traffic flow models frequently apply methods known from hydrodynamics, which basically apply the theorem of continuity. Vehicles, which enter a temporal-spacial element on a link (such as a segment of a road) will have to leave this element if no entrances and exits occurr.

Macroscopic traffic flow models are also based on the fundamental diagram describing the relationship between traffic flow, traffic density and mean speed. All of these macroscopic models need a speed-density curve as input value. Intensive measurements were carried out worldwide to fit the speed-density curves against empirical data on various types of roads emphazising on motorways. All empircal data indicate a monotonically decreasing function. In uncongested situations drivers tend to travel at their desired speed which corresponds with the legal speed varying by the type of road and the national jurisdiction. At about 150 vehicles per lane and km all vehicles are queued without any consideration of the type of road. The empiral data is approximated using linear, quadratic, exponential or polynomial functions.

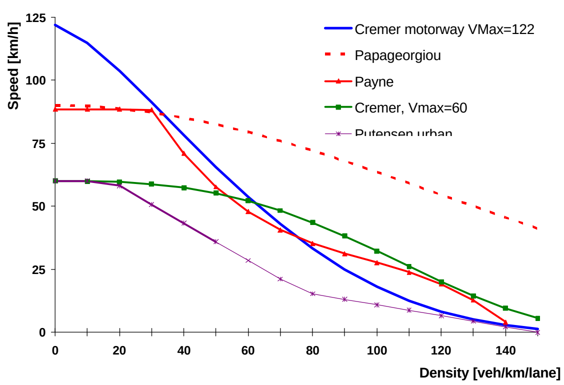

Fig. 3. Speed-density curves proposed by various authors based on specific empirical data

If all drivers would strictly behave in correspondance with the fundamental diagram, they had to react instantaneously to any change in traffic flow. A change in density by incoming traffic at an interchange for example would immediatly reduce mean speed at that point. However the change of speed is not instanteneous and drivers react after a certain delay. Thus modellers proposed in the seventies and eighties a series of macroscopic traffic flow models based on the fundamental diagram and introducing properties of diffusion, inertia and relaxation.

Let us assume that the road (a link) is divided into segments i of length Δ_i (200-500m). Time is devided into fixed steps of length Δt (1-10s). The reaction time is t. $U(k)$ represents the stationary speed-density relationship used as an input value. Then the mean speed $u_i(t+1)$ at segment i in time step $t+1$ is calculated by

$$u_i(t+1) = u_i(t) + (U(k_i(t)) - u_i(t))/t$$
$$+d(k_{i+1}(t) - k_i(t))/\Delta_i$$

The speed at the next time step is calculated from the mean speed in the previous time step plus a relaxation term which pushes the speed into the direction according to the speed-density curve. The third term reflects the anticipation with d being the anticipation distance.

Cremer and others introduced additional terms to handle speed variations between adjacent segments and the introduction of inflows and outflows at entrances and exits.

The advantage of discrete macroscopic models is computational simplicity. Since individual vehicles are unknown in the basic macroscopic models no analysis can be done on values which require single vehicle data. Travel time analysis for example is valuable for a range of applications. Further refinements to the macroscopic models are needed one being implemented in DYNEMO (Schwerdtfeger, 1984).

In uncongested traffic conditions drivers are likely to drive the speed they intend to drive. This speed is not the same for all drivers. Thus the desired speed depends on individual preferences. As density increases less possibilities are available for individual driver behaviour and the speed distribution will shrink. According to the fundamental diagram traffic flow will have a maximum depending on the type of road. At this point q_{max}, the optimum of speed v_{opt} and density k_{opt} are reached thus giving $q_{max} = v_{opt} \bullet k_{opt}$. At densities lower than k_{opt} the desired speed of each vehicle varies while it is assumed to be the same at densities above k_{opt}. While u_i denotes the mean speed in segment i according to the function $U(k)$, v_i denotes the speed of an individual vehicle.

At each time step k_i is updated from a count of the vehicles within the segment. According to the relation U_j the mean speed $u_i = U_j(k_i)$ is computed. A vehicle which is at time t at position $x(t)$ in segment i and drives with speed $v(t)$ is moved in accordance with the speeds u_i and u_{i+1} (the speed in the next section downstream) and with its own state:

$$v(t+\Delta t) = f\,(1 - {}^{x(t)}\!/_{\Delta_i})\; u_i \;+\; {}^{x(t)}\!/_{\Delta_i}\; u_{i+1})$$

The function f selects a speed based on the mean speed $U_j(k_i)$ and the value of the desired speed distribution.

If a vehicle leaves a segment during one simulation step, it is placed in the succeeding segment downstream, and the vehicle counts for both segments are adjusted. Then the traffic densities are updated and the next time step is computed.

For each junction within a network signal control or priority rules are specified separately. Signal control is modelled by inputting green time ratios and maximum discharge rates. If the ratio between green time and discharge rate is lower than the incoming flow rate a queue will build up. Priority rules are modelled by comparison of the flow rate ratio of the major and minor movements.

3.2 Applications of macroscopic modelling

Macroscopic traffic flow models are applied for studies such as impact assessment of incidents on traffic flow. Adding pure macroscopic models with properties to handle single vehicles as done in DYNEMO additional areas of applications arise.

Design of control algorithms for route guidance systems is well suited for simulation. Supplementing the road network by local or temporal decision points, vehicles are able to alter their calculated routes as provided by individual or collective route guidance systems. Depending on the present traffic conditions throughout the entire network, the cost values of each link will change. The cost functions typically depend on the vehicle type again to deal with different trip purposes and other reasons of variety. Only vehicles of certain vehicle types (such as technically equipped cars) are able to receive updates of the current traffic conditions. These vehicle are getting rerouted as better shortest routes might occurr while travelling. Fixed routes without alternatives are specified to reflect fixed detours or signed routes for drivers without mapping information.

Since transport is frequently blamed as one of the major source of environmental pollution planners are entitled to search for remedial measures. Keeping up traffic flow at a certain and stable level will help to reduce pollution. Schemes of traffic control and variable message sign systems are applied. Simulation is applied to test scenarios which are best suited to find optimal solutions. Therefore emission models are coupled with traffic flow models to gain a better knowledge of traffic related emissions in networks of considarable size. To run

During the traffic flow simulation in each simulation time step state information about each vehicle is recorded including current speed and distance travelled since the origin of the trip. This state information, together with the vehicle type and the type of the link on which the vehicle is currently travelling, is used to determine the current emission of the vehicle.

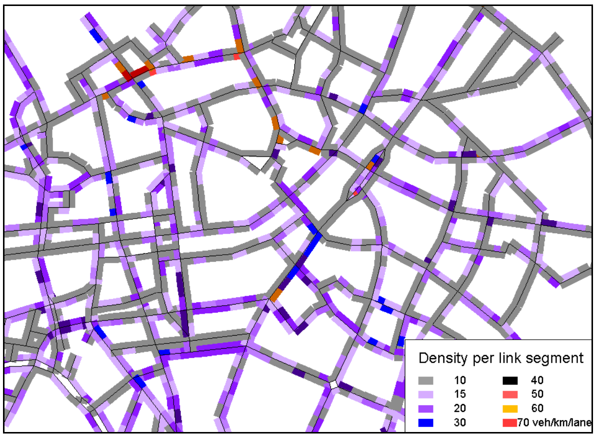

Fig. 4. DYNEMO indicating traffic density [veh/h/lane] at each link segment in central Berlin at 15:30

4 MICROSCOPIC TRAFFIC FLOW SIMULATION

4.1 Mathematical background

A different approach to model traffic is microscopic traffic flow modelling. Interactions of single vehicles depend on driver behaviour and technical features of the vehicles. A family of microscopic models are based on car-following theory by Gazis (1961). Wiedemann (described in Leutzbach, 1988) expanded the model by incorporating also perceputual factors representing individual driver abilities. This model is presented as follows:

The traffic flow model is a discret, stochastic, time step based (originally 1s) microscopic model, with driver-vehicle-units (DVU) as single entities. The model contains a psycho-physical car following model for longitudinal vehicle movement and a rule-based algorithm for lateral movements. As a faster vehicle approaches a slower vehicle on a single lane he has to decelerate. The action point of conscious reaction depends on the speed difference, distance and driver dependend behaviour. Figure 5 indicates the oscillating process of this approach.

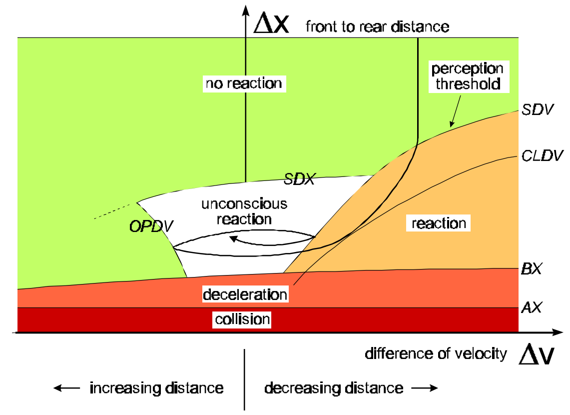

Fig. 5. Car-following model of Wiedemann Thresholds and one vehicle trajectory

The thresholds of figure 5 are explained in an abbreviated form. Driver specific perception abilities and individual risk behaviour is modelled by adding random values to each of the parameters as shown for AX.

AX: Desired distance between the fronts of two successive vehicles in a standing queue $AX := VehL + MinGap + RND1 \cdot AXMult$ with RND1 normally distributed N(0.5, 0.15)

ABX: Desired minimum following distance which is a function of AX, a safety delta distance BX and the speed $ABX := AX + BX \cdot v$

SDV: Action point where a driver consciously observes that he approaches a slower car in front. SDV increases with increasing speed differences (*Dv*). In the original work of Wiedemann an additional threshold cldv (closing delta velocity) is applied to model additional deceleration by usage of the brakes with a larger variation than SDV.

OPDV: Action point where the following driver notices that he is slower than the leading vehicle and starts to accelerate again. The variation of OPDV is large.

SDX: Perception threshold to model the maximum following distance which is about 1.5 - 2.5 times ABX.

A following driver reacts to a leading vehicle on up to a certain distance which is about 150 m. The minimum acceleration and deceleration rate is set to be 0.2 m/s². Maximum rates of acceleration depend on its technical features which are usually lower for trucks than the personal desire of its driver. The model includes a rule for exceeding the maximum deceleration rate in case of emergency. This happens if ABX is exeeded.

The values of the tresholds depend on the present speed of the vehicle. Recent measurements indicate that the original values of the behavioural thresholds have to be adjusted since drivers as a statistical group became more used to driving at higher speeds leading to increasing macroscopic values such as maximum traffic flow (capacity) and higher densities at comparable mean speeds.

By reducing the time step simulated traffic flow will be in better accordance with empirical data than the originally proposed 1s time step. Figure 6 depicts a vehicle as it decelerated due to a traffic light. After a delay of about 15 s the vehicle accelerates again. Using 1s time steps the vehicle decelerates harder than commonly observed in reality which will lead to accelerations in the next second and so on. A reduction to time steps of 1 second indicate a much smoother deceleration phase.

Gipps (1981) proposed a different car following using only two stages of different driving behaviour instead of the four stages

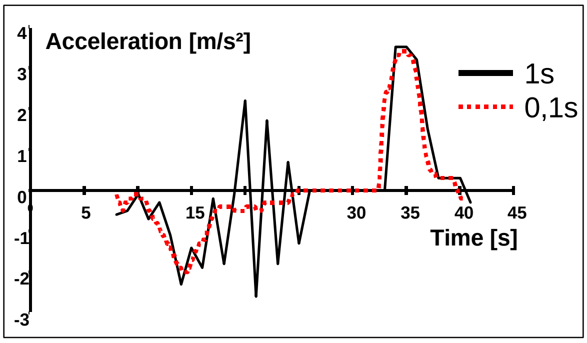

Fig. 6. Car-following thresholds used in urban situations as a function of the speed

Another family of microscopic models were introduced to traffic flow simulation based on cellular automaton theory. Though the original cellular automaton is not well suited to replicate the stochastic effects of individual driver behaviour recent advances were made by supplementing the models using random acceleration and deceleration capabilities of single vehicles (Krauß, 1997).

4.2 Applications of microscopic modelling

Though microscopic traffic flow models tend to be computationally very demanding certain areas of interest can be seen.

Testing the impact of technical equipment such as automatic car following schemes on traffic flow and capacity, which requires detailed car models to handle various technical components of the vehicles.

Feasibility studies of junction layout including the design of the lane allocation and the type of junction such as interchanges, signalized or roundabouts.

Detailed signal control optimizing the type of vehicle actuated signal control at a local controller level or within networks.

Design and operation of public transport in a shared environment with cars as done in many urban areas.

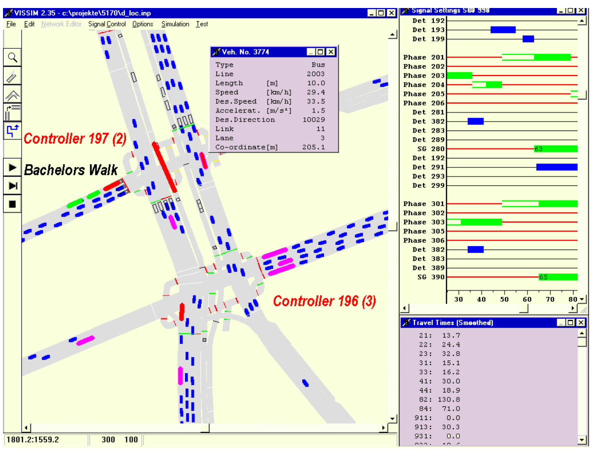

Fig. 7. VISSIM: Traffic flow of individual vehicles to optimize an Urban Traffic Control System called SCATS

5 CONCLUSION AND FUTURE RESEARCH

This paper has indicated three types of models to represent and analyze traffic: i. equilibrium assignment, ii. macroscopic traffic simulation based on the hydrodynamic theory of traffic and iii. microscopic traffic simulation based on car following models of individual vehicles. Each type of models serves different needs and application purposes. All three types lend itself to 'what-if' scenario analyses. Prior to running a simulation the user selects the model to suit his purpose best, the input parameters and the scenario to be studied.

Recently telematic applications in the field of transport are of growing interest as deployment initiatives are launched in Europe, North America and Japan. Models are needed to justify and forecast the benefit of these telematic measures prior to implementation. Future research is needed to integrate the various types of models. Integrated models are required for urban and regional transport taking into account the future requirements of planners, traffic engineers and politicians. Integrated planning models have to be able to predict changes in demand and flow over time to be suited for applications such as

- online traffic monitoring,
- collective route guidance,
- road pricing and
- pretrip information.

As more traffic data will be available due to fast innovations in information technologies, transport and traffic models should keep up to predict benefits of the new technologies in terms of more efficient travel, reduced congestion, less pollution and fewer accidents.

This will probably lead to

- Models which make efficient use of the increasing amount of traffic data being available online
- Static network planning models which will converge to dynamic models as travel choices correlate with traffic responsive control measures
- Microsimulation models which are available for larger networks

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